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Analysis and Optimization of Spinal Codes over the BSC: from the AoI Perspective

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***Abstract*—In the ultra-reliable low-latency communications (URLLC) and the prospective 6G communications, the optimization of the age of information (AoI) will enhance the performance in the real-time status update situations. Spinal codes is a new type of rateless codes which can achieve the channel capacity over both the additive white Gaussian noise (AWGN) channels and the binary symmetric channels (BSC), so minimizing the AoI of Spinal codes will significantly decrease the latency of the real-time status update system. In this paper, we firstly study the AoI of a specific code—Spinal codes, and derive the upper bound of the AoI of Spinal codes. We also prove that obtaining a fine-grained rate in the transmission pattern will decrease the AoI of Spinal codes. Then we formulate the optimizing problem and derive that the incremental-tail-transmission pattern of Spinal codes is the optimal pattern to minimize the AoI. Simulation results demonstrate that the upper bound of the AoI of Spinal codes is tighter when the channel condition is better and the incremental-tail-transmission pattern of Spinal codes is the optimal pattern to achieve the lowest AoI compared with the puncure-based pattern and the pass-to-pass pattern.**

***Index Terms*—Spinal codes, age of information (AoI), transmission pattern.**

I. INTRODUCTION

In the ultra-reliable low-latency communications (URLLC) [11], the age of information (AoI) is often considered as an index to measure the performance of the real-time status update system [1–3]. AoI measures the time from the sender generating the status update to the receiver succeeding in receiving it. Specifically, if the latest status update generated at the time $U(t)$ is received at the time t, then the AoI of the update is $A(t) = t - U(t)$. The URLLC system is one of the three main applications of 5G, and the prospecting 6G will also attach great importance to it. With more and more data transmitted and status updates generated and received in the near future, AoI optimization will decrease the latency of transmission and enhance the experience of real-time status update users.

Recently, there have been a lot of research corresponding to the AoI optimization in the physical layer. [8] and [9] respectively study the AoI of a coding system under the hybrid automatic repeat-request (HARQ) scheme and the Markov decision process (MDP) scheme, and both derive the conclusion that the transmitter should update the status when the AoI reaches a certain threshold. In [14], the author derives the expression of AoI of a linear group code, and simulation results of the AoI of the low density parity check (LDPC) codes with different parameters are given, but no further analysis of the AoI expression is provided. In [12], the expression of the AoI of the random linear codes is

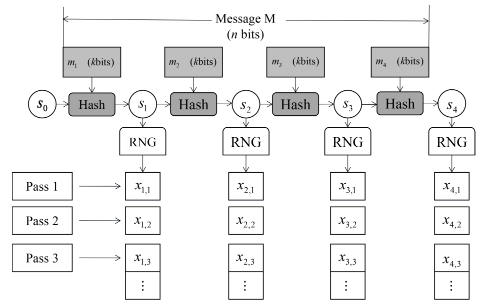

Fig. 1. The encoding process of Spinal codes.

derived, and the analysis of the code scheme optimization, or the ”refinement” of the code block size, is specifically given. These works have made great contributions in the AoI optimization of the coding system. However, until now few previous works pay much attention to the AoI of some specific codes, such as polar codes or Spinal codes. Indeed, optimizing the AoI of some specific codes, such as Spinal codes which has good rate performance, will significantly and fundamentally decrease the latency of the status update system compared with optimizing that of the network layers.

Spinal codes [4, 5] has been proved to reach the channel capacity over the additive Gaussian white noise (AWGN) channel and the binary symmetric channel (BSC) [6]. The key of Spinal codes, as is depicted in Fig. 1, are the hash function and the random number generator (RNG), which can produce pseudo-random code symbols as many as needed. From the aspect of real-time status update with usually short code length, Spinal codes is adaptive in the code rate without considering the channel condition, so it can be applied in the situations where the channel conditions are extremely bad or in other words, extreme URLLC systems. In the coming 6G era, optimizing the AoI of Spinal codes from the perspective of its transmission pattern will further enhance its reliability and the transmission latency of status updates.

In this paper, we combine the transmission pattern characteristics of Spinal codes and the AoI expression derived in [12], and give some further conclusions of the AoI expression. We derive that for Spinal codes over the BSC, the transmission pattern of each round will have an impact on the AoI. Specifically, we prove that for Spinal codes, the AoI is strictly decreasing with the probability of decoding success in each round. Then we take the frame error ratio (FER) upper bound into consideration and derive the upper bound and the limit of

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the AoI of Spinal codes. Based on the upper bound of the AoI, we formulate an optimization problem and give the solution algorithm. Simulation results show that the upper bound of the AoI is tighter when the crossover probability of the BSC is low, and the incremental-tail-transmission pattern takes an advantage and achieves the minimum AoI.

The main contributions of this paper can be summarized as follows:

- We analyze the AoI of a specific code, Spinal codes. To the best of our knowledge, this is the first research of the AoI analysis corresponding to a specific channel codes.
- On the basis of the derived AoI expression, we analytically show that transmitting less symbols in each round, or specifically, puncturing is a way to decrease the AoI.
- By formulating an AoI minimization problem, we obtain that for Spinal codes, incremental-tail-transmission pattern achieves the least AoI compared to the pass-to-pass transmission pattern and the puncture-based pattern.

The rest of this paper is organized as follows. Section II introduces the basic of Spinal codes. In Section III, the AoI upper bounds of Spinal codes are derived. The formulation of the problem and solving algorithm are presented in Section IV. In Section V, simulation results are provided, followed by conclusions in Section VI.

II. Preliminaries

A. Encoding and Decoding Process of Spinal Codes

Different from LT codes and Raptor codes, Spinal codes use hash function and random number generator (RNG) to code messages as infinite pseudo-random symbols. As depicted in Fig.1, the encoding process of Spinal codes contains 4 main steps:

1) An n-bit data frame (or message) M is divided into n/k blocks, with each block containing k bits. The block message is called M_i, $i = 1, 2, ...n/k$.
2) The hash function h receives two inputs: a k-bit message m_i and a v-bit hash value s_{i-1}, and generates one output: a v-bit value s_i, usually $s_0 = 0^v$, where h is: $\{0,1\}^v \times \{0,1\}^k \rightarrow \{0,1\}^v$
   Note that the transmitter and receiver both know the first spine value s_0.
3) The hash value s_i serves as the seed of RNG, and generates c-bit code symbols where RNG is: $\{0,1\}^v \times \mathrm{N} \rightarrow \{0,1\}^c$
   We also denote $x_{i,j}$ as the output symbols.
4) The code symbols will be mapped to a channel input to adapt to the channel characteristics, where f is : $x_{i,j} \rightarrow \Omega$, and usually in the BSC, $f = 1$, which means the code symbols generated from RNG will be straightly sent to the channel.

The transmitter will continuously generate code symbols and send them pass-by-pass during each round, until the symbols are sufficient to be successfully decoded. The reasons why Spinal codes is capacity-achievable are the pairwise-independent characteristic of the hash function and pseudo-random characteristic of the RNG, which result in huge difference of code symbols between different messages.

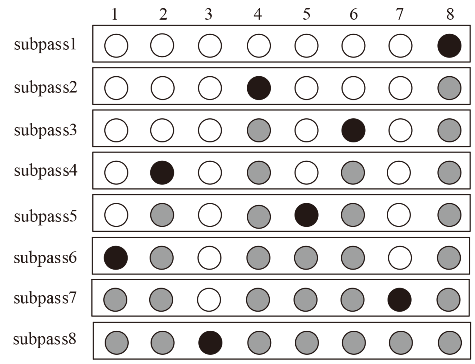

Fig. 2. The uniform puncturing with $\overrightarrow{g} = [8, 4, 6, 2, 5, 1, 7, 3]$. In each subpass,the sender transmits the symbols marked by black circles while those gray circles represent symbols that have already been sent in previous subpasses.

The optimal decoding algorithm of Spinal codes is the maximum likelihood decoding (ML decoding), where the decoder finds the encoded messages that have the least difference with the received code symbols. We denote the vector of the received symbols as $\overline{y}$, and the vector of the encoded message symbols $\overline{x}(M)$ for the message M, then the ML rule of Spinal codes over the BSC is

$$\hat{M} \in \underset{M' \in \{0,1\}^n}{\arg\min} \|\overline{y} \oplus \overline{x}(M')\|. \tag{1}$$

That is, to search for the message M' whose encoded symbols have the smallest Hamming distance with the received symbols.

ML decoding is the optimal decoding method, but its complexity increases exponentially with message length, so ML decoding is only applied in short-code scheme. Long message symbols are usually decoded by bubble algorithm which has lower complexity.

B. Puncture-based Transmission Scheme of Spinal Codes

We have talked about the transmission scheme of pass-by-pass sending. In [5] the author remarked that the rate performance is proportional to $\frac{k}{l}$, where k is the segment parameter and l is the number of the total pass when the message get decoded, but only increasing k will result in high decoding complexity. Puncturing is a trade off of the decoding complexity and the rate performance.

Fig. 2 shows a puncturing scheme of Spinal codes with $n = 8, k = 2$. Each pass is divided into 8 subpasses. In subpass i, the black-colored symbols will be transmitted, with the gray-colored representing the symbols sent before. In this case, the decoding success will happen in any subpass. Although the transmission pattern changes, the decoding algorithm of ML or bubble still works. For the skipped subpasses, the decoding cost will be considered as 0.

Besides the uniform puncturing scheme above, we can also take the non-uniform puncturing scheme, where symbols that correspond different spine values are sent different times. In [13], the author proposes a triangle-shaped puncturing scheme, and simulation results prove that this transmission pattern will enhance the rate performance of Spinal codes compared to the uniform puncturing scheme.

III. Upper Bounds of the AoI for Spinal Codes

In this section, we firstly analyze the monotony of the AoI expression of Spinal codes. Then we will give the upper bound of the frame error ratio (FER) of Spinal codes, which are our previous works. Based on these we derive the upper bound of the AoI of Spinal codes, which will serve as the objective function in the next section. We also prove that a fine-grained rate set will decrease the AoI of Spinal codes.

A. Analysis of the Monotony of the AoI Expression

An incremental-redundancy code can be called $\{n_1, n_2, ...n_m\}$, if data frames are coded and initially n_1 bits are transmitted, and will experience at most (m-1) rounds of incremental transmission if it cannot be successfully decoded. We denote the code length that has been transmitted before round i (round i included) as $n_i, i = 1, 2, ...m$, and the probability that the data frame can be successfully decoded before round i (round i included) $P_s(n_i)$, so we can easily get that $P_s(n_i) = 1 - P_e(n_i)$, with $P_e(n_i)$ representing the FER of the code at round i. For Spinal codes, let d be the code length of one pass, and $c = 1$, so we can get that $d = \frac{n}{k}$. For the pass-to-pass transmission pattern, at the first round, the transmitter send P passes, and if the message cannot be decoded, then an extra pass will be incrementally sent until the total code length reaches n_m. In other words, the rounds will be no more than $m - 1$, so we can easily know that $n_i = (P + i - 1)d, i = 1, 2, ...m$, then according to [12], the expression of the average AoI is

$$\overline{A} = -\frac{1}{2} + d(\frac{P + m - 1 - \sum_{i=1}^{m-1} P_s(n_i)}{P_s(n_m)} + \frac{(P + m - 1)^2 - \sum_{i=1}^{m-1} (2P + 2i - 1)P_s(n_i)}{2(P + m - 1 - \sum_{i=1}^{m-1} P_s(n_i))}). \tag{2}$$

For the puncture-based transmission pattern, when the data frame cannot be decoded at a certain round, we only incrementally send code symbols that correspond a certain spine value instead of the whole pass. Let $c = 1$, which means that in each round only one symbol is incrementally transmitted. In this case, the expression of the average AoI is

$$\overline{A} = -\frac{1}{2} + \frac{N + m - 1 - \sum_{i=1}^{m-1} P_s(n_i)}{P_s(n_m)} + \frac{(N + m - 1)^2 - \sum_{i=1}^{m-1} (2N + 2i - 1)P_s(n_i)}{2(N + m - 1 - \sum_{i=1}^{m-1} P_s(n_i))}, \tag{3}$$

where $N = Pd$.

Analyzing the two expressions, we can derive some theorems as below.

Theorem 1. (Monotony of the AoI) $\overline{A}$ is strictly decreasing with $P_s(n_i) \in (0, 1), i = 1, 2, ...m$.

Proof. Due to the limited space we will only give the proof for (3) and the method is the same for (2).

We will consider the monotony of the simplest $\{n_1, n_2\}$ code. The expression of AoI in this situation will be

$$\overline{A} = -\frac{1}{2} + \frac{N + 1 - P_s(n_1)}{P_s(n_2)} + \frac{(N + 1)^2 - (2N + 1)P_s(n_1)}{2(N + 1 - P_s(n_2))}, N = Pd. \tag{4}$$

For the two terms related to $P_s(n_i)$, $P_s(n_1)$ both serve as the numerator with negative coefficient, so $\overline{A}$ is decreasing with $P_s(n_1)$. Then we consider a dynamic transmission where codes are sent round by round, that is

$$\{n_1, n_2, ...n_m\} \rightarrow \{n_1, n_2\} + \{n_2, n_3\} + ... + \{n_{m-1}, n_m\}. \tag{5}$$

In this case, the multiple-round transmission can be treated as several two-round transmissions. For each two-round transmission $\{n_i, n_{i+1}\}$, we can prove that $\overline{A}$ is decreasing with $P_s(n_i)$. If the $(m + 1)$th transmission is processed, then $\overline{A}$ will be also decreasing with $P_s(n_m)$ So A is decreasing with $P_s(n_i), i = 1, 2, ...m$ in $(0, 1)$. □

B. The Upper Bound and the Limit of the AoI of Spinal Codes

From Theorem 1 we can know that a high probability of a decoding success in each round will fundamentally decrease AoI. Furthermore, if we can derive some conclusions about the FER of Spinal codes, then the AoI or its bound of Spinal codes can be calculated. Now we will give the FER upper bound of Spinal codes.

Lemma 1. (*Our Previous works*) (FER upper bound of Spinal codes over the BSC) Consider the Spinal codes over the BSC with message length n, segmentation parameter k, modulation parameter c and crossover probability f, then the FER under ML decoding can be upper bounded by[15]

$$P_e \leq 1 - \prod_{i=1}^{n/k} (1 - \epsilon_i^{upper}(T_i, U_i)), \tag{6}$$

with

$$\epsilon_i^{upper}(T_i, U_i) = \sum_{t=0}^{T_i} \left\{ \binom{T_i}{t} f^t (1 - f)^{T_i - t} \times \min\left[1, (U_i - 1) \sum_{k=0}^{t} \binom{T_i}{k} 2^{-T_i}\right]\right\}, \tag{7}$$

where $T_i = L(n/k - i + 1)$, $U_i = 2^{k(n/k - i + 1)}$.

From lemma 1 we will know that the probability of a decoding success has a lower bound. Combining Lemma 1 and Theorem 1, we can derive the upper bound of the AoI of Spinal codes over the BSC.

Theorem 2. (The upper bound of the AoI of Spinal codes over the BSC) Consider the Spinal codes over the BSC with message length n, segmentation parameter k, modulation parameter c and crossover probability f. We denote the upper bound of FER of spinal codes as $P_e(i), i=1,2,...m$, where i is the transmission round, then the AoI of Spinal codes under ML decoding can be upper bounded by

$$\begin{aligned}\overline{A} < &-\frac{1}{2} + d(\frac{P+\sum_{i=1}^{m-1} P_e(i)}{P_e(m)} \\ &+ \frac{(P+m-1)^2 - \sum_{i=1}^{m-1}(2N+2i-1)(1-P_e(i))}{2(P+\sum_{i=1}^{m-1} P_e(i))}),\end{aligned} \tag{8}$$

for the pass-to-pass transmission pattern and

$$\begin{aligned}\overline{A} < &-\frac{1}{2} + \frac{N+\sum_{i=1}^{m-1} P_e(i)}{P_e(m)} \\ &+ \frac{(N+m-1)^2 - \sum_{i=1}^{m-1}(2N+2i-1)(1-P_e(i))}{2(N+\sum_{i=1}^{m-1} P_e(i))},\end{aligned} \tag{9}$$

where $N=Pd$ for the puncture-based transmission pattern, and $P_e(i)$ has been given in (6) and (7).

From Theorem 1 we can not only derive the upper bound of the AoI, but also calculate the limit when the channel is so ideal that no error will happen in the transmission. According to the theorem below we can know that the limit is only the function of the code length in the first round.

Theorem 3. (The limit of AoI) When $P_s(n_i) \to 1, i = 1,2,...m$, we will derive that the AoI upper bound has a limit of $\lim_{P_s(n_i)\to 1} \overline{A} = -\frac{1}{2} + \frac{3}{2}N$.

Proof. According to the monotony of $P_s(n_i), i=1,2,...m$, let $P_s(n_i)=1$,then the the limit will be reached, and we can derive the result of the AoI limit. □

Remark 1. For the situation when a whole pass is incrementally sent if a frame cannot be decoded, the limit is similarly $\frac{3}{2}Pd$. The proof is the same as Theorem 3.

We have derived the expression of the AoI of Spinal codes under two incremental transmission pattern, which are the puncture-based pattern and the pass-to-pass pattern. If we take the uniform puncture pattern, where $n=8$, $k=2$, $c=1$, $\vec{g}=[g_1,g_2,g_3,g_4]$ where g_1,g_2,g_3,g_4 are different from each other. We will prove that the former pattern will have lower AoI.

Theorem 4. (Puncture-based pattern have lower AoI than pass-to-pass pattern) Consider Spinal codes with message length n, segment parameter k and modulation parameter 1, let the code length of a pass $d=n/k$. Initially N code symbols are transmitted, and if decode fails, (1) d symbols will be incrementally sent under uniform puncture pattern in the next d rounds, or (2) they are incrementally sent as a whole pass in the next round. Let the average AoI of situation (1) be $\overline{A_1}$ and situation (2) $\overline{A_2}$, then we have $\overline{A_1} < \overline{A_2}$.

Proof. We take $n=8$ and $k=2$ as example, and the same is for other values of n. We first write the code pattern according to [12] as $[N, N+1, N+2, N+3, N+4]$ and $[N, N+4]$ for (1) and (2) respectively. Obviously $P_s(n_1) < P_s(n_2) < P_s(n_3) < P_s(n_4)$ because of the characteristics of Spinal codes, so we can calculate their difference

$$\begin{aligned}\overline{A_1} - \overline{A_2} = &\frac{3P_s(n_1) - \sum_{i=2}^{4} P_s(n_i)}{P_s(n_5)} \\ &- \frac{(N+4)^2 - 4(2N+4)P_s(n_1)}{2(N+4-4F(n_1))} \\ &+ \frac{(N+4)^2 - \sum_{i=2}^{4}(2N+2i-1)P_s(n_i)}{2(N+4-\sum_{i=2}^{4} P_s(n_1))} \\ &< \frac{3P_s(n_1) - P_s(n_2) - P_s(n_3) - P_s(n_4)}{P_s(n_5)} \\ &+ \frac{3P_s(n_1) + P_s(n_2) - P_s(n_3) - 3P_s(n_4)}{2(N+4-4P_s(n_1))} < 0.\end{aligned} \tag{10}$$

□

Remark 2. Theorem 4 implies that transmitting less codes in each round will decrease the AoI. In the next section, we will set the step of code length as 1 in the optimization problem to get a lower AoI minimum.

IV. AoI Optimization of Spinal Codes

In this section, we will optimize the transmission pattern of Spinal codes with the minimum AoI, using the upper bound of AoI derived in Section III. We first formulate the problem as an integer programming problem and give the solution algorithm. We observe that under ML decoding and with the total code length of the data frame restricted, if the code length of the first round is fixed, the minimum AoI can be reached by the incremental-tail-transmission pattern. Based on this conclusion, we again study the minimum AoI under the incremental-tail-transmission pattern with the fixed total code length. We derive that the less code symbols are transmitted in the first round, the less AoI Spinal codes will reach.

A. Problem Formulation and Solution

We aim to minimize the AoI of Spinal codes under the restriction of total code length. From Section III we know that the AoI is decreasing with the probability of decoding success in each round, so the most important parameter of the AoI of Spinal codes is the transmission pattern variation during each round, which directly affects the FER of Spinal codes.

For Spinal codes over the BSC with message length n and segment parameter k, N symbols are sent in the first round, and one symbol will be incrementally transmitted in the subsequent rounds with the total code length no more than $L_{\max}$, then we can formulate the problem of AoI optimization as:

Problem 1. Find the total pattern $P=[p_1,p_2,p_3,...p_{\frac{n}{k}}]$ that:

$$\min \overline{A} \\ s.t. \begin{cases} N,m \in Z^+ \\ N+m-1 < L_{\max} \end{cases} \tag{11}$$

It is obviously a linear integer programming, and easy to solve it by the iteration algorithm. In each round, we will choose the symbol that will achieve the lowest AoI to transmit, by choosing the smallest FER of the patterns according to the monotony of the AoI in Theorem 1, until the total code length reaches L_max. The algorithm that solves the problem can be described as Algorithm 1.

Algorithm 1 The optimal transmission pattern of the AoI of Spinal codes

$P=[r,r,...,r]$, $N=r\cdot\frac{n}{k}$, $m=0$, $f=10^{-3}$.
while $N+m<L_{\max}$ **do**
  Update the incremental round: $m \leftarrow m+1$
  for $i \leftarrow 1$ to n/k **do**
    Update the decision variable:$P_i \leftarrow P_i+1$
    Calculate FERUpperBound(i) under the crossover probability f and store them
    restore the decision variable: $P_i \leftarrow P_i-1$
  end for
  Search for the minimum of FERUpperBound and get the index d
  $P_d \leftarrow P_d+1$
end while
end

Solving the problem, we will get the transmission pattern that has minimum AoI. We take $n=8, k=2, c=1, r=3$ as example, and use Algorithm 1 to solve, and the results are shown in Table I.

TABLE I
THE OPTIMAL TRANSMISSION PATTERN OVER THE BSC

Total code length $L_{\max}$	Decision variables
20	$L=[3,3,3,11]$
24	$L=[3,3,3,15]$
28	$L=[3,3,3,19]$
32	$L=[3,3,3,23]$

From Table I, we can conclude that continuously transmitting the code symbols from the $\frac{n}{k}$th spine value, or in other words, incremental-tail-transmission pattern, will result in the lowest AoI.

We have derived the optimal transmission pattern—incremental-tail-transmission, but in Algorithm 1 the initial code length is fixed. In fact, the code length of the first round also affects the AoI according to Theorem 3, so the initial passes of symbols can also be optimized. We can write the optimization problem as

Problem 2. Find the total pattern $P=[p_1,p_2,p_3,...p_{\frac{n}{k}}]$ that:

$$\min \overline{A} \\ s.t. \begin{cases} N,m \in Z^+ \\ N+m-1 = L_{\max} \\ N=ki, i=1,2,...L_{\max}/\frac{n}{k} \end{cases} \tag{12}$$

The algorithm below will search for the minimum AoI with the total code length restricted under the incremental-tail-transmission pattern. Solving the problem, we derive that

Algorithm 2 The algorithm to optimize the AoI in initial passes under incremental-tail-transmission pattern

$L_{\max}, f=f_0$
for $r \leftarrow 1$ to $L_{\max}/\frac{n}{k}$ **do**
  $N \leftarrow r\cdot\frac{n}{k}, m \leftarrow L_{\max}-r\cdot\frac{n}{k}, P \leftarrow [r,r,r,...,r]$
  $P_{\frac{n}{k}} \leftarrow P_{\frac{n}{k}}+m$
  Calculate AoIUpperBound(r) under the pattern P
end for
Find the AoIUpperBound minimum and get the index d
end

when $d=1$, which means only transmit one pass in the first round, the minimum AoI will be reached.

Remark 3. Algorithm 2 does not take the feedback delay and the transmission delay into consideration. If the ACK/NACK feedback latency is considered, the least initial passes could not be the optimal solution.

V. SIMULATION RESULTS

In this section, some simulation results of the AoI will be presented to verify the upper bound of the AoI of Spinal codes which has been derived in Section III. Moreover, the AoI of Spinal codes under different transmission pattern will be compared to further prove the solution given in Section IV.

Fig. 3 compares the average AoI simulation and the AoI upper bound of Spinal codes derived in section 3.Concretely, we choose $n=8$, $k=2$, $c=1$ in the simulation to apply ML decoding algorithm. We take the pass-to-pass transmission pattern where initially 5 passes of symbols are transmitted and the total passes of symbols are no more than 14. According to Fig. 3, the AoI upper bound in this case is tighter when the crossover probability is low. But the difference between the upper bound and simulation results is high when the channel error probability is high, because in this case lower correct probability in some rounds will enlarge the AoI upper bound, and correspondingly its difference from the simulation results.

Fig. 4 compares the average AoI simulation of Spinal codes under three different transmission pattern: pass-to-pass pattern, puncture-based pattern and the incremental-tail-transmission pattern. Specifically, we choose $n=8$, $k=2$, $c=1$, and at the first round 3 passes of code symbols are transmitted. According to Fig. 4, the AoI simulation results of the three

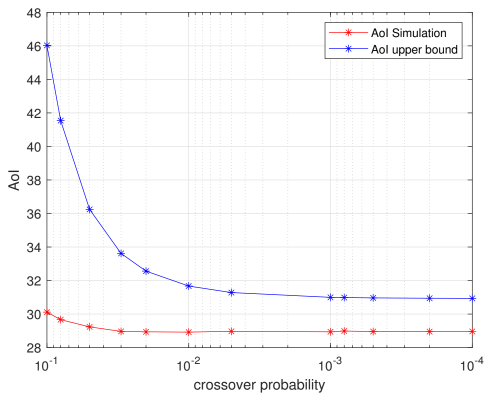

Fig. 3. Comparison between the simulation results and the upper bounds of the AoI of Spinal codes with $n = 8$ and $k = 2$.

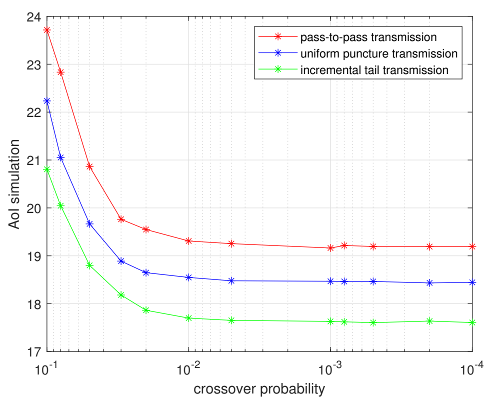

Fig. 4. Comparison among the simulation results of the AoI of Spinal codes under three transmission patterns with $n = 8$ and $k = 2$.

transmission patterns have a large difference among themselves. The pass-to-pass pattern achieves the highest AoI, and the incremental-tail-transmission pattern achieves the lowest AoI among the three patterns. Besides, the puncture-based transmission pattern takes the second place, proving that a fine-grained rate set will decrease the AoI of Spinal codes.

VI. Conclusions and Future Work

In this paper, we have derived the upper bound of the AoI of Spinal codes, and prove that transmitting less codes in each round will significantly decrease the AoI of Spinal codes. Then we formulate and solve the AoI optimization problem, concluding that incremental-tail-transmission pattern is the optimal pattern to achieve the lowest AoI of Spinal codes, and the less the initial passes are, the lower AoI Spinal codes will achieve. In the future, the time AoI of the systems with transmission delay and feedback delay will be studied, and also the joint source-channel coding with Spinal codes and other block codes combined will be researched.

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