Partial Self-Concatenation Structure and Performance Analysis of Spinal Codes Over Rayleigh Fading Channel

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***Abstract*—Spinal codes are a new family of rateless codes which have been proved to be capacity-achieving over both the additive white Gaussian noise (AWGN) channel and the binary symmetric channel (BSC). However, over the Rayleigh fading channel between the vehicle nodes, the error performance of Spinal codes is not satisfactory since the tail message blocks of Spinal codes are prone to error. To solve this issue, this paper proposes a new practical partial self-concatenation coding structure of Spinal codes, named N tail-protected Spinal codes, to pointedly protect tail message blocks of Spinal codes over the flat Rayleigh fading channel. Theoretical performance analysis and simulation results are also provided to validate and verify the effectiveness of the proposed N tail-protected Spinal codes.**

***Index Terms*—Spinal codes, fading channel, performance analysis, maximum likelihood (ML) decoding, bubble decoding.**

I. Introduction

In the 5 G wireless communications, channel fading is a critical obstacle to realize Ultra-Reliable and Low-Latency Communication (URLLC). Specifically, the quality of service over the Internet of Vehicles (IoV) is affected by the high speed of vehicles and the multi-path propagation of electromagnetic waves. In order to ensure reliable transmission over wireless channels between vehicle nodes, channel codes including fixed-rate codes and rateless codes are applied as an essential technique to mitigate the impact of fading. However, accurate estimation for channel state information (CSI) is strongly needed for fixed-rate codes, but in IoV scenario, the number of vehicles varies rapidly due to the traffic, which leads to less accurate CSI estimation and less moderate rate setting.

Rateless codes, unlike the fixed-rate codes, are more adaptive and less dependent to CSI, because they can adjust the transmission rate to the dynamic fading channels. Spinal codes [1] are a new class of rateless codes that have been proved to achieve the Shannon capacity over both the additive white Gaussian noise (AWGN) channel and the binary symmetric channel (BSC) [2]. Since Spinal codes are proposed, there are many previous works on the structure improvement of Spinal codes over AWGN channel, BSC or erasure channels, such as employing two-way coding scheme to improve rate performance [3], applying unequal error protection to achieve lower frame error rate (FER) [4][5], compressing the sparse source to get higher throughput [6] and proposing more dynamic decoding algorithm with memory to decrease decoding complexity [7]. However, few works focus on the structure design of Spinal codes over fading channels, which restricts its application in urban vehicular networks. To the best knowledge of the authors, the only existing works on this topic are [8] where a trust function decoding algorithm is proposed to increase the decoding accuracy, and our previous work [9], where we design a complete self-concatenation structure for Spinal codes to improve the FER performance over the Rayleigh fading channel. Nevertheless, a significant disadvantage of the complete self-concatenation structure is that it can only be applied in extremely short messages, which is insufficient for practical application in vehicular networks. Efficient coding structure for Spinal codes with long message over Rayleigh fading channel is still to be invented. In this paper, a new coding structure for Spinal codes complementary to the complete self-concatenation Spinal codes is invented and studied, which can be applied under longer message length. Instead of self-concatenating all the message blocks in the complete self-concatenation structure, the new structure self-concatenates only several tail message blocks, forming a structure of partial self-concatenation. According to the number of self-concatenated tail message blocks, this partial self-concatenation structure can be called as N tail-protected structure, where N is a variable integer. Specifically, we give the encoding structure and decoding algorithms of this new structure for Spinal codes, and theoretically analyze its FER performance over the Rayleigh fading channel by deriving the FER upper bound of N tail-protected Spinal codes. Simulation results demonstrate that the new structure outperforms original Spinal codes regarding the error and rate performance over the Rayleigh fading channel, at a cost of little complexity increase compared to original structure.

The rest of the paper is organized as follows. Section II gives the encoding structure, decoding algorithms and complexity analysis of the N tail-protected Spinal codes. The FER upper bound for the proposed N tail-protected Spinal codes is derived in Section III. Simulation results are given in Section IV, and the paper is concluded in Section V.

II. N Tail-Protected Spinal Codes

A. Encoding Process of N Tail-Protected Spinal Codes

The encoding process of N tail-protected Spinal codes is similar to original Spinal codes and the complete self-concatenation Spinal codes, both consisting of the hash function and the random numeral generator (RNG). However, the generation processes of the spine values and code symbols are slightly different from the other two structures. Specifically, as depicted

Manuscript received December 5, 2021; revised February 19, 2022; accepted March 21, 2022. Date of publication March 24, 2022; date of current version June 24, 2022. This work was supported in part by the National Natural Science Foundation of China under Grants 61871147 and 62071141, and in part by the Shenzhen Municipal Science and Technology Plan under Grant GXWD20201230155427003-20200730122528002. The review of this article was coordinated by Dr. Songnam Hong. *(Corresponding author: Shaohua Wu.)*

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Digital Object Identifier 10.1109/TVT.2022.3162018

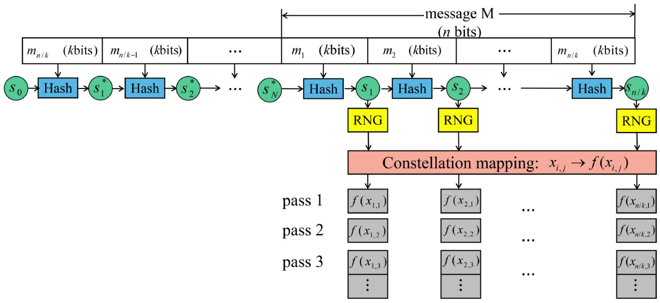

Fig. 1. The encoding structure of N tail-protected Spinal codes.

in Fig. 1, the encoding process of N tail-protected Spinal codes contains 4 main steps:

1) *Dividing and concatenating:* An n-bit source message M is divided into n/k blocks, with each block m_i containing k bits. Then the N tail message blocks $[m_{n/k-N+1},\ldots,m_{n/k}]$ are inverted and concatenated before the first message block m_1. After the concatenation, the message $M=[m_1,\ldots,m_{n/k}]$ turns to $M'=[m_{n/k},m_{n/k-1},\ldots,m_{n/k-N+1},m_1,m_2,\ldots,m_{n/k}]$.
2) *Generating the spine values:* The hash function h maps a k-bit message and a v-bit spine value to a v-bit spine value. Specifically, the hash function will firstly map the concatenated tail blocks, that is $s_1^*=h(s_0,m_{n/k})$ and $s_i^*=h(s_{i-1}^*,m_{n/k-i+1}), i=2,3,\ldots,N$. Then the value s_N^* will serve as the input of next hash function to iteratively map the original blocks, that is $s_1=h(s_N^*,m_1)$ and $s_i=h(s_{i-1},m_i), i=2,3,\ldots,n/k$. Usually $s_0=0$ is chosen to initialize the mapping, which are known by both the transmitter and the receiver.
3) *RNG mapping:* The spine value $s_i, i=1,2,\ldots,n/k$ serves as the seed of RNG to continuously generate c-bit encoded symbols $x_{i,j}$ pass by pass, where j denotes the pass of the symbol.
4) *Channel constellation mapping:* The encoded symbols will be mapped by constellation f to the modulated symbols transmitted over the channel, that is $x_{i,j}\to f(x_{i,j})$.

We consider a classic system model in the urban vehicular mobile communication, where the modulated code symbols $f(x_{i,j})$ will experience the time-variant Rayleigh fading and additive white Gaussian noise, that is $y_{i,j}=r_{i,j}f(x_{i,j})+\nu_{i,j}$, where $r_{i,j}$ is the fading coefficient for $x_{i,j}$ with Rayleigh parameter σ_1^2, and $\nu_{i,j}$ is the Gaussian noise with variance σ^2.

For original Spinal codes, because of the serial encoding $s_i=h(m_i,s_{i-1})$ and the effect of spine values on the encoded symbols, $x_{i,j}$ contains information of the blocks $m_a, a\le i$, so the decoding error of the block m_i can result from the error of the encoded symbols $x_{a,j}, a\le i$. Consequently, the tail message blocks with larger a are more error-prone than the front ones. However, for N tail-protected Spinal codes, N tail blocks are concatenated in front of the original message. The encoded symbols $x_{1,1},x_{1,2},\ldots$ contain the information of these tail blocks. Only when m_1 gets wrong does the concatenated tail blocks fail in decoding, which occurs with lower probability. In the new coding structure, the tail message blocks are *protected* by the first block and its encode symbols in the aspect of lower error probability, which is the source of the name N *tail-protected Spinal codes.* The error probability of each message block will be analyzed in Section III.

B. Decoding Algorithms of N Tail-Protected Spinal Codes

N tail-protected Spinal codes are tree codes with $N+n/k$ layers where all the branch nodes of the tree are utilized. In the perfect 2^k-ary coding tree of Spinal codes, each node stores a 2^k-bit possible message block and its corresponding spine value. Identical to the complete self-concatenated Spinal codes, the maximum likelihood (ML) decoding algorithm of N tail-protected Spinal codes searches for the candidate message whose encoded symbols have the least difference with the received symbols layer by layer. Denote the received symbols as $\bar{y}$, and the encoder output of candidate message M as $\bar{x}(M)$, then the ML rule of N tail-protected Spinal codes is

$$\hat{M}\in\arg\min||\bar{y}-\bar{x}(M)||^2, M\in\{0,1\}^n. \quad (1)$$

For the Rayleigh fading channel, $||\bar{y}-\bar{x}(M)||^2$ is in fact the Euclidean distance between the encoded symbols of the candidate message and the received symbols.

The challenge of ML decoding algorithm derives from its exponentially increasing complexity with the message length. In a more general application scenario with long messages, bubble decoding algorithm is more efficient [1]. In the bubble decoding algorithm of N tail-protected Spinal codes, the coding tree of the candidate messages is constructed and pruned layer by layer. In each layer, the father nodes will spread 2^k candidate child nodes, and the path cost is calculated in each child node. If the total number of nodes is larger than pruning depth B, then only B nodes with the least path costs will remain. The process of spreading and pruning will continue until the $(N+n/k)$th layer, and the node path with the least path cost, also the optimal candidate message will be output. The bubble decoding algorithm of N tail-protected Spinal codes is elaborated in Algorithm 1.

The differences of the bubble decoding algorithm for N tail-protected Spinal codes from original Spinal codes and complete self-concatenation Spinal codes are as follows. Firstly, the calculation of path cost of N tail-protected Spinal codes is different. The N self-concatenated blocks output no encoded symbols, so in the first N layers, all the nodes have a zero path cost. In the subsequent layers, the path cost of a node from the ith layer is calculated by the branch cost $||\bar{y}_i-\bar{x}(m_i)||^2$ plus the path cost of its father node, where $\bar{y}_i$ and $\bar{x}(m_i)$ denote the received symbols corresponding to the ith spine value and candidate encoded symbols of ith spine value, respectively.

Secondly, the pruning depth for N tail-protected Spinal codes should be moderate. For original Spinal codes the pruning depth can be arbitrarily selected. However, for the N tail-protected structure the pruning depth is restricted. Specifically, in each layer of the tree, the number of nodes is $2^{ki}, i=1,2,\ldots n/k+N$ without pruning. Because there are 2^{kN} nodes with zero path cost in the Nth layer, pruning before it will possibly remove the correct node path. So the process of pruning must start from the $(N+1)$th layer, or in other words, the pruning depth B should be larger than or equal to 2^{kN}. In this paper, we choose $B=256$ for both N tail-protected Spinal codes and original Spinal

Algorithm 1: The Bubble Decoding Algorithm of N Tail-Protected Spinal Codes.

Input: n, k, tail protection length N, received symbols $\bar{y}$.
Output: The node path with the least path cost.
Initialize pruning depth B, root node $c_{0,0}$;
for $i \leftarrow 1$ *to* $n/k + N$ **do**
  Spread the node in the current layer i. Specifically, for each father node $c_{i-1,j}, j = 1, 2, ..., \min\{2^{k(i-1)}, B\}$, 2^k candidate messages are hash-mapped and stored by child node $c_{i,j}, j = 1, 2, ..., 2^k$;
  if $i > N$ **then**
    The node $c_{i,j}$ generates candidate encoded symbols $\bar{x}_i(M)$ by RNG and calculate $||\bar{y}_i - \bar{x}_i(M)||^2$ and the path cost of node $c_{i,j}$;
  if *the number of nodes* $> B$ **then**
    B nodes with least path cost $c_{i,j}, j = 1, 2, ..., B$ will remain and be spread in the next cycle;
return The optimal node path;

codes to equally compare their performance, which satisfies the condition $B \geq 2^{kN}$.

C. Complexity Analysis of N Tail-Protected Spinal Codes

ML decoding algorithm is processed over the whole coding tree with 2^{Nk+n} leaf nodes in the $(N + n/k)$th layer, so the complexity of ML decoding algorithm is $O(2^{Nk+n})$.

In the bubble decoding algorithm of N tail-protected Spinal codes, hash and RNG mapping is executed in all the $N + n/k$ layers. Let l be the number of passes, and we can derive that in each layer $B2^k$ hashes and $Bl2^k$ RNG mapping is executed. However, the calculation and comparison of path cost are only executed in n/k layers whose nodes have non-zero path cost, with each layer at most $B2^k$ times of comparisons. So the complexity of bubble decoding algorithm is $O((n/k + N) \cdot Bl2^k)$ hashes plus $O((n/k) \cdot B2^k)$ comparisons. Note that the N tail-protected structure for Spinal codes is a refinement and implementation of the complete self-concatenation structure for Spinal codes, because the decoding algorithms of complete self-concatenation structure is impractical for long messages due to their extremely high complexity. The self concatenation Spinal codes with parameters n and k can be considered as n/k tail-protected Spinal codes, with a complexity of $O(2^{2n})$ for ML decoding and $O(2n/k \cdot Bl2^k) + O((n/k) \cdot B2^k)$ for bubble decoding with $B \geq 2^n$, which is a huge decoding cost for the decoder.

However, for long message length n and small N under bubble decoding, the complexity of N tail-protected Spinal codes is close to and slightly larger than that of original Spinal codes [1] and much lower than that of self concatenation Spinal codes. Table I compares the time of running 10 frames of N tail-protected Spinal codes and original Spinal codes, with $n = 256$, $k = 4$, $B = 256$. It is demonstrated from Table I that the concatenated N blocks increase little in the aspects of decoding complexity.

TABLE I
COMPARISON OF THE TIME OF RUNNING 1,000 FRAMES WITH DIFFERENT N, WHERE $n = 256$, $k = 4$, $B = 256$

N	**The time of running 10 data frames (s)**
0	931.583
1	950.285
2	971.045

III. FER PERFORMANCE ANALYSIS FOR N TAIL-PROTECTED SPINAL CODES OVER THE RAYLEIGH FADING CHANNEL

In our previous work [9], based on the analysis of union bound of error probability of each message block, the FER upper bound of original Spinal codes over Rayleigh fading channel is derived. Inspired by the proof of the Theorem 1 in [9], we derive the following FER upper bound for the N tail-protected Spinal codes.

Theorem 1: **(The upper bound of the FER for N tail-protected Spinal codes over the Rayleigh fading channel)** Consider N tail-protected Spinal codes over the Rayleigh fading channel with message length n, segmentation parameter k, modulation parameter c, noise variance σ^2 and the Rayleigh parameter σ_1^2. Let l_i be the number of encoded symbols generated from the ith spine value, then decoded by the ML algorithm, the FER can be upper-bounded as follows:

$$P_e \leq 1 - \prod_{a=1}^{n/k} \left\{1 - \min\left(1, (2^k - 1)2^{n-ak} \min\left(1, R_a\right)\right)\right\}, \tag{2}$$

where

$$R_a = \begin{cases} \frac{\left(\frac{\pi(1+\varepsilon)g(\sigma,\sigma_1)b_a}{2^{2c}}\right)^{b_a/2}}{\Gamma(1+b_a/2)}, & 1 \leqslant a \leqslant n/k - N \\ \frac{\left(\frac{\pi(1+\varepsilon)g(\sigma,\sigma_1)b_1}{2^{2c}}\right)^{b_1/2}}{\Gamma(1+b_1/2)}, & n/k - N + 1 \leqslant a \leqslant n/k. \end{cases} \tag{3}$$

where $b_a = \sum_{i=a}^{n/k} l_i$, $g(\sigma, \sigma_1) = P(2{\sigma_1}^2 - \sqrt{2\pi}\sigma_1 + 1) + \sigma^2$, and $P = (2^c + 1)(2^c - 1)/12$ is the power of the code symbol.

Proof: We divide the proof into 3 parts. Firstly, we classify all the candidate sequences into the correct sequence and other wrong sequences. Then we analyze the cost of the correct sequence. Finally we analyze the cost of wrong sequences and further derive the error probability of a wrong sequence.

1) *Candidate Sequences Classification:* Denote the n-bit message as $M = [m_1, m_2, \ldots, m_{n/k}]$. Let $x_{i,j}(M)$ be the jth encoded symbol corresponding to the ith spine value s_i of message M, $y_{i,j}$ be the corresponding received symbol at the receiver, and $\nu_{i,j}$ be the additive white Gaussian noise. Over Rayleigh fading channel with coefficient $r_{i,j}$ we can easily get that $y_{i,j} = r_{i,j}x_{i,j} + \nu_{i,j}$, and the ML decoding algorithm is

$$\hat{M} = \underset{M' \in \{0,1\}^n}{\arg\min} \sum_{i=1}^{n/k} \sum_{j=1}^{l_i} \|y_{i,j} - x_{i,j}(M)\|, \tag{4}$$

where M denotes the candidate sequences.

We classify the candidate sequences into two subsets, the correct sequence $\mathcal{M}_c = \{m_c | m_c = M\}$ and the other wrong sequences $\mathcal{M}_w = \{m_w | m_w \neq M\}$.

2) *Cost Analysis of Correct Sequences:* We calculate the cost of the correct sequence $D(m_c)$, also the distance between the encoded symbols of the correct sequence and the received code symbols, that is

$$D(m_c) = \sum_{i=1}^{n/k} \sum_{j=1}^{l_i} (r_{i,j} x_{i,j}(M) + \nu_{i,j} - x_{i,j}(M))^2. \quad (5)$$

Assume that the Rayleigh fading parameter r is independent and identically distributed (i.i.d), we have

$$\mathbb{E}(r_{i,j} x_{i,j}(M) + \nu_{i,j} - x_{i,j}(M))^2$$
$$= \frac{1}{12}(2\sigma_1^2 - \sqrt{2\pi}\sigma_1 + 1)(2^c+1)(2^c-1) + \sigma^2 = g(\sigma, \sigma_1). (6)$$

From (6) we can easily derive the expectation of $D(m_c)$, that is $\mathbb{E}(D_a(m_c)) = g(\sigma, \sigma_1) \sum_{i=a}^{n/k} l_i$. By applying the Chernoff bound, we can assert that the probability of the event $D_a(m_c) \leqslant (1+\varepsilon) g(\sigma, \sigma_1) \sum_{i=a}^{n/k} l_i$ is nearly 1 when n is large. More detailed derivation of this conclusion can be seen in [9].

3) *Cost Analysis of Wrong Sequences:* Finally we calculate the cost of wrong sequences denoted by $D(m_w)$, and the main difference of the proof from that in [9] lies here. According to ML criterion, when a frame gets wrong, the cost of a certain wrong sequence is less than that of correct one. Denote the event $\mathcal{M}_e$: $\exists m_w \in \mathcal{M}_w$, such that $D(m_w) \leqslant D(m_c)$, we can calculate the FER as $P_e = \mathbb{P}(\mathcal{M}_e)$.

Again, we denote the conditional error probability of the ath block as $\epsilon_a = \mathbb{P}(E_a|\overline{E_1}, \ldots, \overline{E_{a-1}})$, and by calculating $D(m_w)$ and comparing the volume of $D(m_w)$ and $D(m_c)$[9], we can derive that for original Spinal codes, we have

$$\epsilon_a \leqslant \min\left(1, (2^k - 1) 2^{n-ak} \min(1, R_a)\right), \quad (7)$$

where

$$R_a = \frac{1}{\Gamma\left(1 + \sum_{i=a}^{n/k} l_i/2\right)} \left(\frac{\pi(1+\varepsilon) g(\sigma, \sigma_1) \sum_{i=a}^{n/k} l_i}{2^{2c}} \right)^{\sum_{i=a}^{n/k} l_i/2}. \quad (8)$$

For N tail-protected Spinal codes, the N concatenated tail blocks are protected, for only the encoded symbols $x_{1,j}, j = 1, 2, \ldots$ will affect their decoding, which are the same as the first block, so the error probability of these blocks is the same as that of the first block which is ϵ_1, and the rest blocks remain the same as original Spinal codes. We denote $b_a = \sum_{i=a}^{n/k} l_i$, then we derive the conditional error probability of ath block of N tail-protected Spinal codes as follows:

$$\epsilon_a \leqslant \min\left(1, (2^k - 1) 2^{n-ak} \min(1, R_a)\right), \quad (9)$$

where

$$R_a = \begin{cases} \frac{\left(\frac{\pi(1+\varepsilon) g(\sigma, \sigma_1) b_a}{2^{2c}}\right)^{b_a/2}}{\Gamma(1+b_a/2)}, & 1 \leqslant a \leqslant n/k - N \\ \frac{\left(\frac{\pi(1+\varepsilon) g(\sigma, \sigma_1) b_1}{2^{2c}}\right)^{b_1/2}}{\Gamma(1+b_1/2)}, & n/k - N + 1 \leqslant a \leqslant n/k. \end{cases} \quad (10)$$

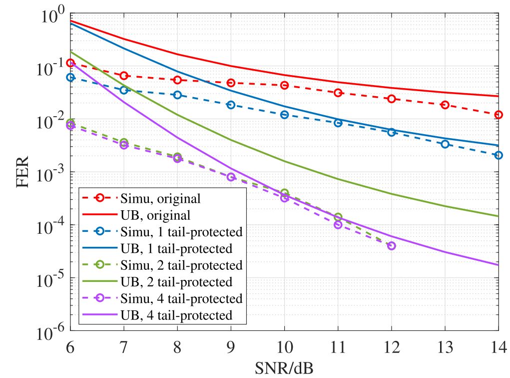

Fig. 2. Comparison of FER simulation and upper bound of N tail-protected/original Spinal codes over Rayleigh channel with $n = 8$, $k = 2$.

The FER can also be expressed as

$$P_e = \mathbb{P}(E_1 \cup \ldots \cup E_{n/k})$$
$$= 1 - \prod_{a=1}^{n/k} (1 - \mathbb{P}(E_a|\overline{E_1}, \ldots, \overline{E_{a-1}}))$$
$$= 1 - \prod_{a=1}^{n/k} (1 - \epsilon_a). \quad (11)$$

Since P_e is increasing with ϵ_a, so by substituting the right side of (9) to (11) we can derive the upper bound of P_e.

From Theorem 1, it is obvious that the FER upper bound of N tail-protected Spinal codes over Rayleigh fading channel is lower than original Spinal codes because of the concatenated tail blocks, and we can further derive that the larger N is, the lower FER the N tail-protected Spinal codes can achieve. However, more concatenation blocks means larger pruning depth, which results in higher decoding complexity, so in the practical coding for long messages we recommend that N be no larger than 2 to achieve a moderate pruning depth.

IV. Simulation Results

In subsection IV-A and IV-B, N tail-protected Spinal codes achieve a fixed rate to observe FER performance, and in subsection IV-C, N tail-protected Spinal codes are considered as rateless codes to observe rate performance. In our simulation, we assume that different code symbols experience different random Rayleigh coefficients and there are cyclic redundancy check bits to decide whether the frame is decoded correctly.

A. Error Performance Bounds Verification

We first analyze the FER upper bound of short message N tail-protected Spinal codes, complete self-concatenated Spinal codes [9] and original Spinal codes under ML decoding. Fig. 2 compares the FER simulation (Simu) and FER upper bound (UB) of N tail-protected Spinal codes, original Spinal codes and complete self-concatenated Spinal codes (i.e. 4 tail-protected Spinal codes) over Rayleigh fading channel, with $n = 8$, $k = 2$, $c = 8$, $\sigma_1 = 0.5$, and the total number of passes is 8. For original Spinal codes and 1 tail-protected Spinal codes, we choose

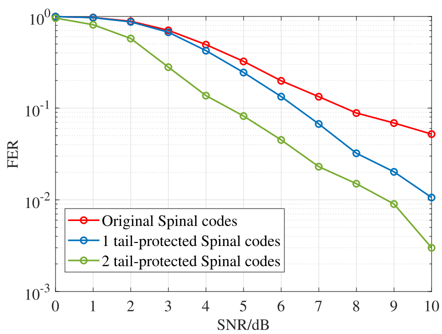

Fig. 3. Comparison of FER performance of N tail-protected/original Spinal codes over Rayleigh fading channel with $n = 256$, $k = 4$.

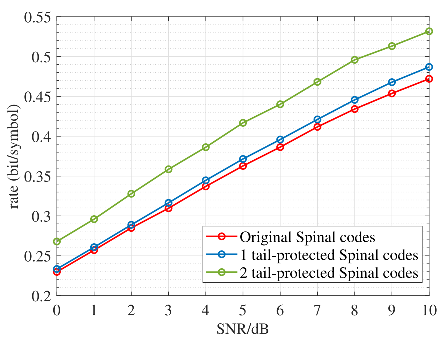

Fig. 4. Comparison of rate performance of N tail-protected/original Spinal codes over Rayleigh fading channel with $n = 256$, $k = 4$.

$\epsilon = 0.1$ in (3); for 2 or 4 tail-protected Spinal codes we choose $\epsilon = 0.01$, so that the upper bound is tight with the simulation results. The error performance and FER upper bound of the new structure can be analyzed from two perspectives. On the one hand, it can be seen from Fig. 2 that the FER upper bound of N tail-protected structure is far lower than that of original Spinal codes, and the FER upper bound decreases as N becomes large, which validates the theoretical analysis. On the other hand, compared to complete self-concatenation Spinal codes, although the FER bound of 2 tail-protected Spinal codes is higher, the simulation results are nearly the same. Fig. 2 implies that several concatenation blocks can achieve the same FER performance as complete self-concatenation structure under short message length.

B. FER Performance Comparison

Fig. 3 compares the FER performance of long message N tail-protected Spinal codes and original Spinal codes under bubble decoding over the Rayleigh fading channel, where $n = 256$, $k = 4$, $B = 256$, $\sigma_1 = 0.5$ and the total number of passes is 12. Fig. 3 shows that each tail block redundancy significantly reduces the FER. At a cost of little complexity increase, the new structure gains much in FER performance, which is the superiority of N tail-protected Spinal codes over original Spinal codes.

C. Rate Performance Comparison

Fig. 4 shows the rate performance of rateless N tail-protected Spinal codes and original Spinal codes over the Rayleigh fading channel under pass-to-pass transmission pattern, where $n = 256$, $k = 4$, $B = 256$ and $\sigma_1 = 0.5$. From Fig. 4, the coding rate of 2 tail-protected Spinal codes increases by about 25% in average compared to original Spinal codes, and about 20% compared to 1 tail-protected Spinal codes, which are significant gains. Considering the FER and rate performance together with the complexity, we recommend that 2 tail-protected Spinal codes be used for long messages.

V. Conclusion and Future Work

In this paper, N tail-protected Spinal codes are proposed as a refinement of complete self-concatenation Spinal codes over Rayleigh fading channel. The new structure decreases the error probability of tail blocks, directly achieving lower FER. We derive the FER upper bound of N tail-protected Spinal codes over Rayleigh fading channel. Simulation results show that 2 tail-protected Spinal codes outperform original Spinal codes in terms of both FER and rate over Rayleigh fading channel. In the future, the hardware implementation of N tail-protected Spinal codes in IoV will be an interesting topic.

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